

Motivation

- FTQC involves deep and complex **cross-layer compilation stacks**
- Logical metrics do not reflect **real execution cost**
- Single-metric optimization is insufficient, FTQC is inherently **multi-objective**
- Optimization depends on different FTQC **protocols and architectures**

FTQC Compilation and Execution Stack

Algorithm

Shor Grover HHL QSP/QSVT QAOA VQE

Logical Circuit / IR

Clifford+T Decomposition **Circuit Optimization** Unitary Synthesis

Fault-tolerant Program Realization

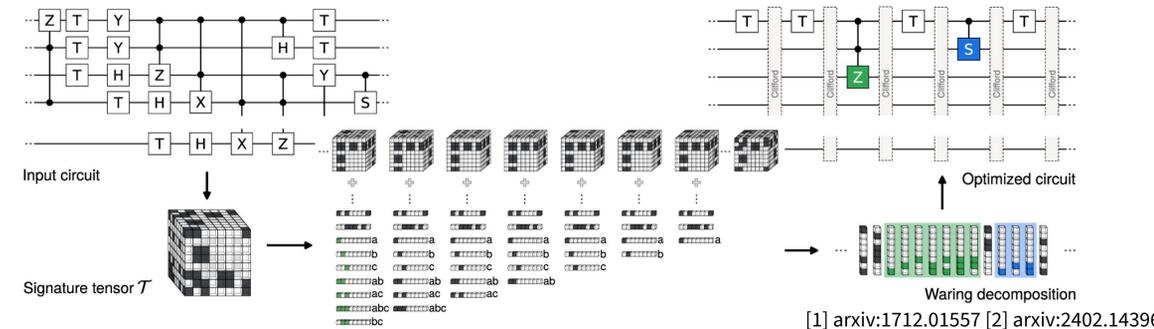
Fault-tolerant Gate Constructions	Architecture-specific Operations	Physical Gate Mappings
Surface Code: Lattice Surgery	Superconducting: Joint Parity Measurement	Surgery / Braiding Scheduling
Color Code: Transversal Clifford	Neutral Atom: Transversal CNOT	Magic State Factory Placement
Code Switching	Trapped Ion: All-to-all Connectivity	Measurement Ordering
Magic-State Cultivation	Photonic:	Routing Resolution

Classical Control

Syndrome Extraction Decoding Feed-forward Correction Pauli Frame Tracking

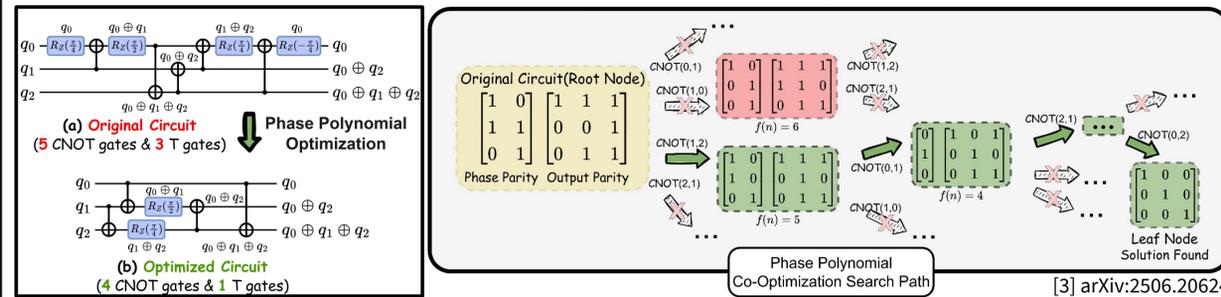
T-only circuit optimization

- Exploit algebraic structure to compress the phase tensor into a lower-rank representation using a decoding-based method[1].
- Use reinforcement learning to search over circuit transformations to discover lower-rank tensor representations(which can lead to fewer T gates)[2].



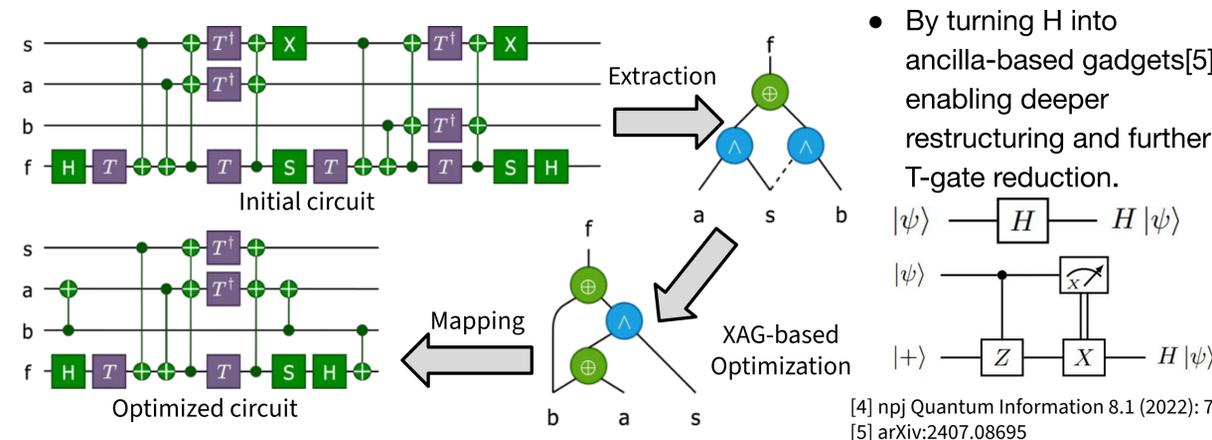
CNOT-only optimization

- CNOT-Phase circuit can be efficiently represented using *Phase Polynomials IR*.
- *PhasePoly*[3] using heuristic search to transform the parity table with fewer steps.

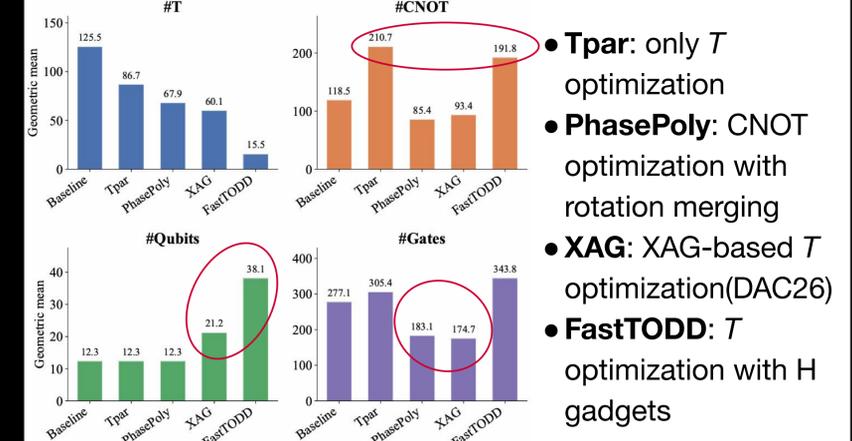


Ancilla-needed optimization

- XAG-based synthesis[4] rewrites circuits into XOR-AND structures to reduce T gates via algebraic simplification and ancilla-assisted decomposition.



Key Observations



- **Tpar**: only *T* optimization
- **PhasePoly**: CNOT optimization with rotation merging
- **XAG**: XAG-based *T* optimization(DAC26)
- **FastTODD**: *T* optimization with H gadgets

Unoptimized baseline consists of 24 diverse circuit benchmarks, each bar shows the Geo Mean for the corresponding metric.

Error-correction protocols and architecture-specific cost model

- Different **QECCs** (surface code, qldbc, color code...)
- Magic State **Factory** vs Magic State **Cultivation**
- **Lattice Surgery** CNOT vs **Transversal** CNOT
- **Ancilla-Rich** vs Qubit-Limited Regimes
- 2D Color Code {**H**, **S**, **CNOT**} vs 3D Color Code {**CZ**, **CCZ**, **CNOT**, **T**}
- **Superconducting** vs Non-Local Architectures (**Ion Trap / Neutral Atom**)

Key Takeaways

- Need **cross-layer IR** to enable **holistic, end-to-end** optimization
- Need **multi-objective** cost modeling beyond single-gate reduction
- Need QECC- and architecture-aware **optimization strategies and cost model**